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Investigation of Cavitating Cascades

Experiments on cavitating and noncavitating cascades were carried out in a conventional water tunnel modified for this purpose. The comparison of the experimental results with theory, in both the fully wetted and fully cavitating conditions, was found to be satisfactory.

Introduction

Present day interest in the high-speed performance of hydrofoil boats, ship propellers, pumps, and turbines has resulted in an intensive study of cavitation in liquid flows. During this process the flow field changes significantly from that of a homogenous single-phase flow. Forces, pressure distributions, and other performance characteristics are then correspondingly altered and these changes must be reflected in the design parameters of machines in which cavitation is to take place. The effects of cavitation on the performance of pumps and turbines are found empirically by testing complete units to determine the change in the output quantities such as head and efficiency as the inlet pressure (or an appropriate cavitation index) is changed. Such tests, though useful and even essential for application, offer little physical insight into the cavitation process itself. Experimental studies have been carried out on isolated cavitating hydrofoils of a type useful for propellers and for hydrofoil boat applications. These studies reveal more of the mechanics of the cavitation process than do overall performance tests although they do not pertain directly to complete turbomachines. As is well known, a cascade or lattice of hydrofoils simulates one important aspect of the flow through axial turbomachines. The present highly developed state of the axial flow compressor owes much to the thorough experimental and theoretical studies of cascade aerodynamics of recent years. Some of the problems of cascade theories and experiment are reviewed in references [1 through 4].[1]

The flow through a cascade is clearly more amenable to analysis and controlled experimentation than is that in a complete machine. As in the case of single-phase flow (through an axial compressor or pump) it seems certain that a study of cavitating cascades will lead to a better understanding of cavitation in turbomachines. Experimental work on cavitating cascades has been, however, sparse and only the work of Numachi [5] is known to the authors. Professor Numachi's work has been mainly concerned with the development of suitable profiles for accelerating and decelerating cascades in which a relatively small amount of cavitation is present on the hydrofoils.

Objectives of the Present Investigation

As there is a relative lack of information concerning cavitating cascades, it was desired to carry out experiments on a representative cascade of simple hydrofoils throughout the entire range of cavitation from inception and partial cavitation to full cavitation or supercavitation. It was also desired to determine whether the unsteady cavitating effects previously observed on isolated hydrofoils occurred in cascade and, if so, whether the phenomenon had the same behavior as the isolated case or would give rise perhaps to a condition similar to that of propagating stall observed in compressor cascades. There are, furthermore, a number of experimental procedural problems that must be solved in carrying out tests on cavitating

Contributed by the Fluids Engineering Division and presented at the Winter Annual Meeting, New York, N. Y., November 27–December 1, 1966, of The American Society of Mechanical Engineers. Manuscript received at ASME Headquarters, July 5, 1966. Paper No. 66—WA/FE-25.

[1] Numbers in brackets designate References at end of paper.

Nomenclature

A = plan form area
c = chord length
C_D = drag coefficient $= \dfrac{D}{A\rho V^2/2}$
C_L = lift coefficient $= \dfrac{L}{A\rho V^2/2}$
C_M = moment coefficient $= \dfrac{M}{Ac\rho V^2/2}$
C_{Dm} = mean drag coefficient $= \dfrac{D_m}{A\rho V_m{}^2/2}$
C_{Lm} = mean lift coefficient $= \dfrac{L_m}{A\rho V_m{}^2/2}$
C_{Mm} = mean moment coefficient $= \dfrac{M}{Ac\rho V_m{}^2/2}$
C_N = normal force coefficient $= \dfrac{N}{A\rho V^2/2}$
D = force parallel to, V_1
D_m = force parallel to vector mean velocity, V_m
f = frequency of oscillations
K_v = cavitation parameter based on vapor pressure $\dfrac{p - p_v}{\rho V_1{}^2/2}$
l = cavity length
L = force perpendicular to, V_1
L_m = force perpendicular to vector mean velocity, V_m
M = moment on model about center of section
N = force perpendicular to the model chord
p = tunnel static pressure
p_v = vapor pressure of water
Re = Reynolds number $= \dfrac{Vc}{\nu}$
s = foil spacing in cascade
t = thickness of hydrofoil
V_1 = upstream tunnel velocity
V_2 = velocity leaving cascade
V_m = vector mean of V_1 and V_2
α = inlet flow angle measured from chord line to V_1
α_2 = downstream flow angle measured from chord line $= (\alpha - \theta)$
α_m = mean angle of attack measured from chord line $= (\alpha - \theta_m)$
β = stagger angle, angle between normal to chord line and cascade axis $= (\gamma - \alpha)$
γ = nominal stagger angle, angle between axis of cascade and normal to upstream velocity
δ = deviation angle, angle between exit flow direction and tangent to camber line at the trailing edge
θ = measured angle between upstream and downstream flow directions
θ_m = angle between V_1 and V_m
ρ = density of water
σ = solidity, ratio of chord to spacing $= c/s$

cascades not normally encountered in air cascade tests. These problems arise because in the usual water tunnel, unlike air cascade tunnels, the flow is not discharged to atmosphere or a large plenum but is made to recirculate in a closed loop. It was necessary then to determine whether or not consistent procedures could be easily developed to simulate cascade conditions throughout the entire range of cavitation. In addition, the questions of side wall and end wall boundary-layer interactions with the main flow arise in cavitating flow tests just as in air tests.

It would indeed be a very ambitious project to undertake cavitation cascade tests of all possible configurations of interest. As indicated previously, the present observations are centered on cavitation phenomena in a typical representative cascade. With the scope of the experiments thus limited it becomes possible to consider adapting existing facilities to the requirements of the work. In the present experiments the two-dimensional test section of the high-speed water tunnel at the California Institute of Technology was used to carry out the cascade tests. In a certain sense, then, one of the aims of the present work is to determine if it is feasible to perform cascade measurements in such a facility. In the sections that follow, the modifications to the tunnel necessary to perform cascade measurements are described and the basic geometry of the cascade and hydrofoil profiles used are discussed. The results of the fully wetted and cavitating experiments are then presented and comparisons are made where possible with similar experiments and appropriate theories.

Experimental Apparatus

The basic circuit, force measuring and pressurization systems of the high-speed water tunnel [6] at the California Institute of Technology were used except that the new two-dimensional working section [7] and attendant transition sections were installed. The dimensions of this working section are 6 in. × 30 in. high and 50 in. in length. To make cascade measurements, two basic modifications to the working section were made: The working section height was reduced from 30 in. to 15.6 in. by two streamlined inserts which terminated at the cascade axis. The flow leaving the cascade can then be allowed to leave as a submerged jet as in air cascade tunnels or adjustable side walls can be used to guide the exit flow. With the present tunnel geometry it was not practicable to incorporate a large enough plenum chamber to allow a submerged jet to mix freely with surrounding liquid and movable walls hinged at the cascade axis were used instead to control the downstream flow angle. These guiding walls terminated at the end of the working section and formed a submerged jet which discharged into the tunnel diffuser.

It was hoped to make the cascade to some extent representative of configurations likely to occur in practice yet within the limitations of the test section. The basic configuration adopted was a cascade of five vanes with a nominal stagger angle of 45 deg and spacing of 3.20 in. The working section is normally closed by a transparent plastic window 3 in. thick. Four of the vanes were cantilever supported in this window and the central vane was mounted on the force measuring balance of the water tunnel. The hydrofoils chosen for investigation were 8 percent thick planoconvex hydrofoils of 4 in. chord. The leading and trailing edges were rounded off to one-half percent of the chord. A simple angle-changing mechanism was attached to the vanes mounted in the window. The apparatus as constructed permitted each of the vanes to be adjusted through ±10 deg and each of the guiding walls could be similarly adjusted through ±10 deg.

The basic cascade then had a solidity of 1.25. The vanes could each be rotated 180 deg so that diffusing cascades of nominal 45-deg stagger angle could be tested or accelerating cascades of nominal −45-deg stagger angle could be investigated. By removing two vanes an effective solidity of 0.625 could be obtained although there are then only three vanes in the cascade.

The cascade section was operated horizontally to avoid a large pressure gradient occurring along the cascade axis. Fig. 1 shows the tunnel and horizontal working section. The final arrangement of the cascade is shown in Fig. 2. A photograph of the hydrofoil used is shown in Fig. 3, and Fig. 4 shows a close-up view of the cascade working section.

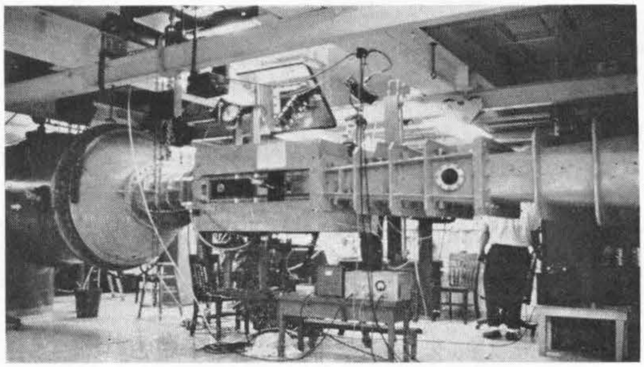

Fig. 1 View of water tunnel working section for cascade studies

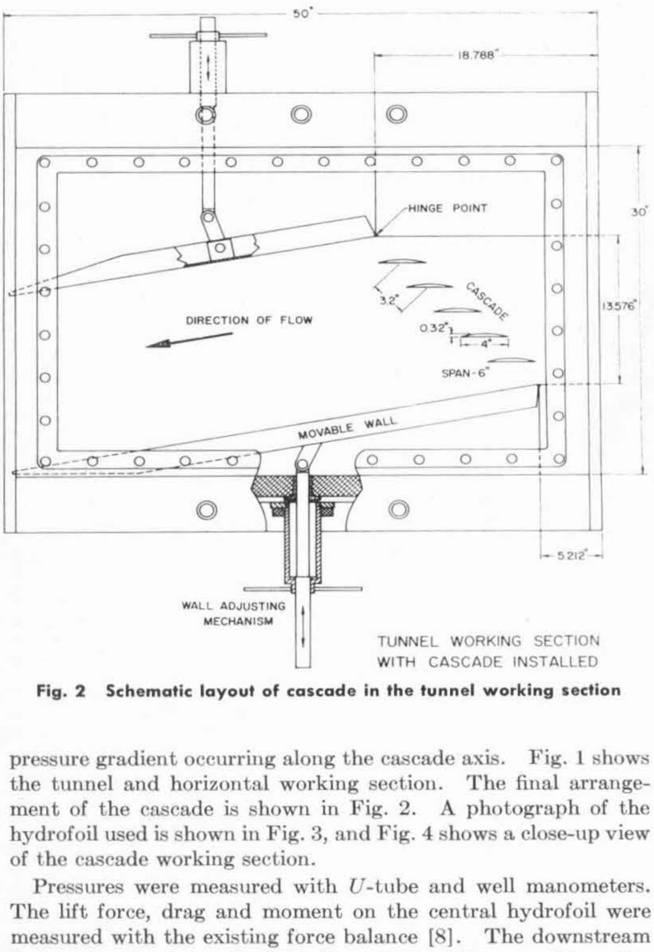

Fig. 2 Schematic layout of cascade in the tunnel working section

Pressures were measured with U-tube and well manometers. The lift force, drag and moment on the central hydrofoil were measured with the existing force balance [8]. The downstream flow angle was determined by the setting of the movable walls; a Conrad-type directional probe was also used to measure the downstream flow angle. This probe was mounted on the center line of the tunnel 6 chords downstream of the cascade axis.

This completes the basic outline of the cascade test arrangement. Noticeably, no provision for either end wall or side wall boundary-layer removal were made. Without rather extensive alterations to the tunnel it would not have been possible to incorporate these features. However, the flow turning and stagger angle are fairly modest so that the effects of side wall boundary-layer growth should not be great.

Experimental Procedure

The velocity approaching the cascade was measured by deter-

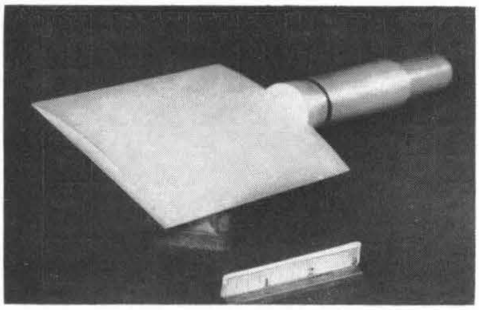

Fig. 3 Photograph of one of the cascade hydrofoils

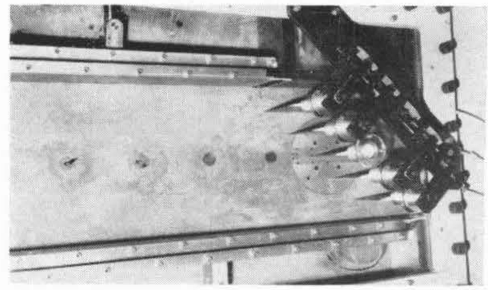

Fig. 4 The final arrangement of the cascade in the water tunnel

mining the pressure differential across the contracting nozzle. The inlet reference pressure was measured eight chord lengths upstream of the center of the cascade. These pressures were related to the pressures throughout the working section by performing a calibration in the clear tunnel. The details of this and other calibration procedures are given in reference [9].

One of the fundamental points to be established in the cascade tests was the question of how best the cascade conditions could be simulated; i.e., what criterion must be used to determine the turning angle of the cascade under test conditions. In aerodynamic cascade work, the use of side wall boundary-layer suction and boundary-layer slots has been shown to improve the two-dimensionality of the flow through the cascade. The amount of suction to apply, however, is determined by matching the measured lift with the measured turning angles according to two-dimensional momentum calculations. In the present tests as in those of Numachi [5] the angle of the movable walls must be adjusted so that the turning angle matches that of the measured forces on the given cascade. This procedure, though, necessarily involves errors arising from neglecting the above side wall boundary-layer effects. The method, however, provides a direct and simple way of conducting the tests. An iteration procedure was used in which successive measured force coefficients are matched with turning angles obtained from momentum calculations assuming two-dimensional flow. The same procedure is used to adjust wall angles in the partially cavitating condition as well. For the fully cavitating flows, however, this system did not have to be used, since it was observed experimentally that the lift force there was fairly independent of the wall setting. It was easy to adjust the wall angles until the cavities from all the foils were of equal length, thus insuring an even pressure distribution across the downstream working section.

The initial program of tests covered the fully wetted and cavitating performance of the cascade arranged as a compressor with a solidity of 1.25. For the 0.625 solidity, two complete tests were conducted at two inlet angles, and finally only fully wetted results were carried through for the turbine configuration at a solidity of 1.25.

The procedure adopted for carrying out the tests in fully wetted flow was as follows: The cascade was set in the particular configuration desired and fully wetted data were taken at a constant tunnel speed and pressure by changing the angle of attack of the hydrofoils. For each data point the wall angles were set according to the scheme already described. For the compressor of 1.25 solidity, the range of inlet angles varied from -2 deg to 6 deg. For this solidity larger angles were not possible owing to the limitation of the wall angle settings.

For cavitating tests, the angle of attack was held fixed and the tunnel static pressure varied at constant tunnel speed to cover the range of cavitation from inception to the fully choked case. There, once again, the walls were adjusted to suit the measured lift and drag for the partially cavitating region. In the fully cavitating region the procedure mentioned earlier was used.

The points of detachment of the cavities in cascade were not predictable. For angles of attack from -2 deg to approximately

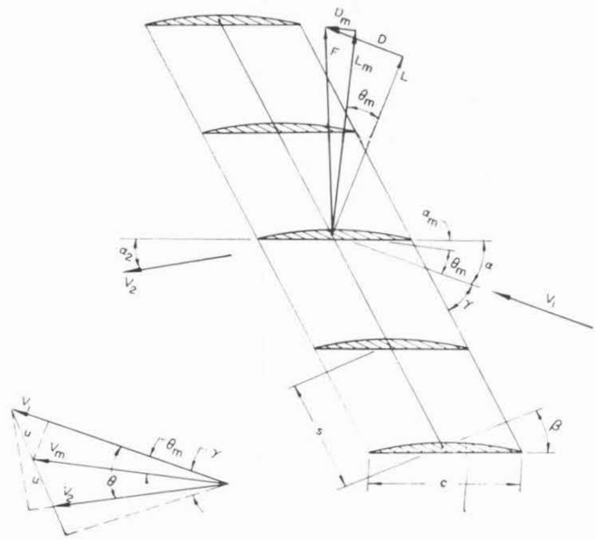

Fig. 5 Definition of cascade parameters and other notation used

6 deg the cavities started at the leading edge of the foils but on the *under* side or flat surface. As these cavities became long, however, some cavitation on the top of the hydrofoils also occurred near the trailing edge. An orifice to measure the pressure within the cavity had been provided on the hydrofoil. For most of the angles of attack used the orifice was not in the cavity but was wetted. It was therefore decided for consistency to base all cavitation numbers on vapor pressure even though it is known that the cavitation number so obtained is slightly high.

Results

We consider first the fully wetted results as these should be comparable with previous experience in aerodynamic cascades. The fully wetted results are presented in terms of force coefficients based on the vector mean velocity through the cascade presuming that the flow is two-dimensional. The downstream leaving flow angle was taken to be that of the movable walls themselves. The cavitating results are, however, based upon the upstream velocity as the mean velocity in cavity flow is not well defined.

1 Fully Wetted Flow. Figs. 6 and 7 give the force coefficients and flow turning for the 1.25 solidity compressor cascade. Also shown are the results of the cascade theories of Schlichting [10] and Mellor [11]. The experimental results fall below those of Mellor's theory by about 11 percent but when account is taken of the variable stagger angle the experimental and theoretical curves are seen to be parallel; that is, the lift slopes are about the same. Similar results were observed for the 0.625 solidity

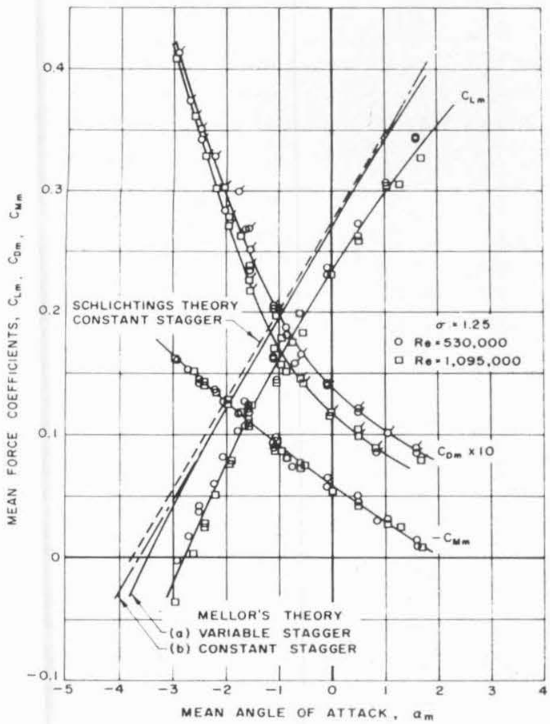

Fig. 6 Force coefficients based on mean flow velocity as a function of the vector mean angle of attack for compressor cascade of solidity σ = 1.25, for two Reynolds numbers

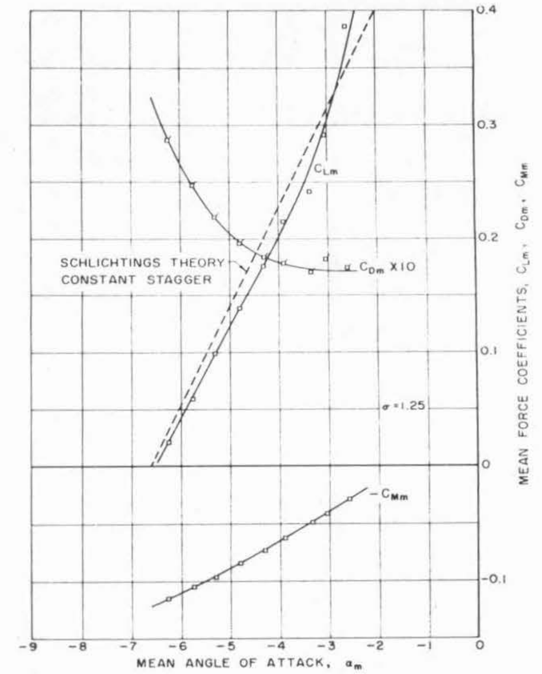

Fig. 8 Force coefficients based on mean flow velocity as a function of the vector mean angle of attack for turbine cascade of solidity σ = 1.25, Reynolds number = 1,095,000.

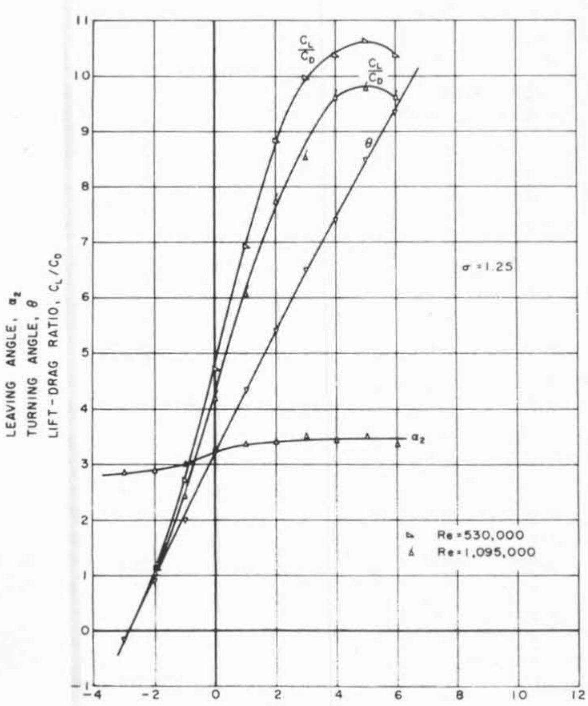

Fig. 7 Lift-to-drag ratio, turning angle and leaving angle as a function of the angle of attack α for compressor cascade of solidity σ = 1.25, for two Reynolds numbers

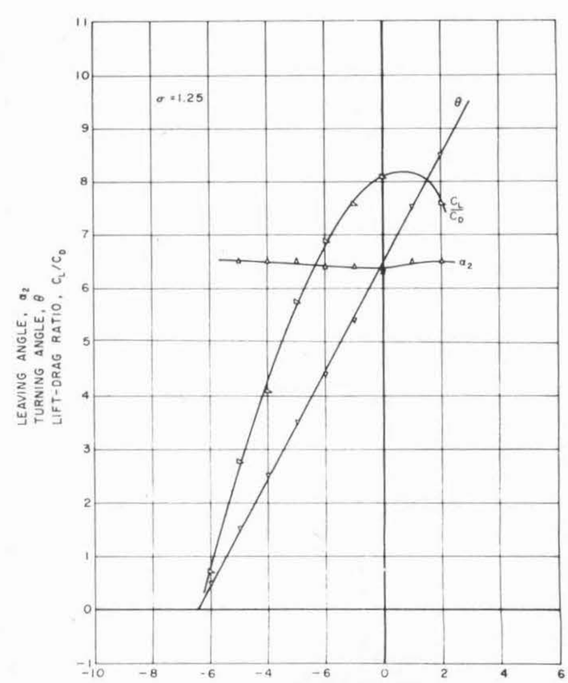

Fig. 9 Lift-to-drag ratio, turning angle and leaving angle as a function of the angle of attack α for turbine cascade of solidity σ = 1.25, Reynolds number = 1,095,000

Table 1

	Solidity σ	Reference [10] $\beta = 45°$	Reference [11] $\beta = 45°$	Reference [11] Varying β	Reference [12]	Exp Varying β
C_{Lm} at $\alpha_m = 0$	1.25	0.270	0.266	0.265		0.233
	0.63			0.74		0.66
α_m for $C_{Lm} = 0$	1.25	$-3°44'$	$-3°36'$	$-3°28'$		$-2°45'$
δ	1.25				$5°11'$	$4°46'$
	0.63				$6°44'$	$7°35'$

compressor cascade. In Table 1 the present results are compared with cascade theory and the semiempirical rule of Constant [12]. It can be seen that there is a reasonable degree of agreement though it is not by any means perfect. However, for this type of hydrofoil, one with a sharp leading edge, the agreement between potential theory and experiment is as good in cascade as it is for an isolated hydrofoil [13].

Similar measurements were carried out in a turbine cascade with a solidity of 1.25 and nominal stagger angle of -45 deg, and these results are shown in Figs. 8 and 9. Also shown is the cascade theory in reference [10] for a constant stagger angle of -45 deg. (It should be pointed out that in the present experiments the stagger angle, β, varies with the inlet angle, α.) The agreement between the theory and experiment is somewhat better than in the compressor cascade as would be expected, owing to the favorable pressure gradient existing across the cascade. The measured deviation angle is within one-third deg of Constant's modified rule [12] for this cascade.

The force coefficients, turning angles and mean angle of attack have been determined using the experimentally found wall angle. This angle was compared with the downstream flow angle as measured by the Conrad directional probe for the 1.25 solidity compressor and turbine cascades. In general, the flow angle measured by the probe was larger by about one-half deg than the wall angle for the compressor cascade and was within one-quarter deg for the turbine cascade. There is, however, always some uncertainty in the flow direction measured by only one probe as the flow may be subject to local influences from upstream wakes or possibly from downstream pressure gradients. Moreover, under cavitating conditions it is not clear that the probe will provide a correct indication of flow direction unless special provisions are made to keep bubbles from entering the probe. For these reasons and to have a single consistent procedure the wall angle itself was used as the downstream flow direction.

2 Cavitating Results. A short description of the physical events occurring when cavitation takes place in the cascade is of interest at this point. As mentioned previously the tunnel speed and inlet angle were fixed and the ambient tunnel pressure was lowered to cause the cavitation. Cavitation usually started at the sharp leading edge either on the pressure side for inlet flow angles less than about 6 deg or on the suction side for angles greater than about 6–7 deg. With decreasing pressure the length of the cavitating zone increased; what is herein termed "partial cavitation" refers to the length of this zone being less than the chord. When the partial cavities became about one third of the chord, periodic oscillations of the cavity and surrounding flow field were observed to occur much like those reported on isolated hydrofoils of the same shape [13]. These oscillations appear to have a definite frequency and to be related to the tunnel speed as in reference [13] but no extensive measurements of these quantities were made. With further reduction of pressure the cavities became longer than the chord and the oscillations ceased. The flow then may be called supercavitating. When supercavitating, the length of the cavity could be varied from that just beyond the chord to the full length of the working section, around seven chord lengths. However, it was difficult to control the tunnel pressure sufficiently precisely to obtain a steady flow for cavity lengths intermediate between those of about a chord in length and those corresponding to the maximum length. The latter condition, when the cavities correspond to maximum length, is often called "choked" flow. The flow was very steady when choked cavity flow occurred.

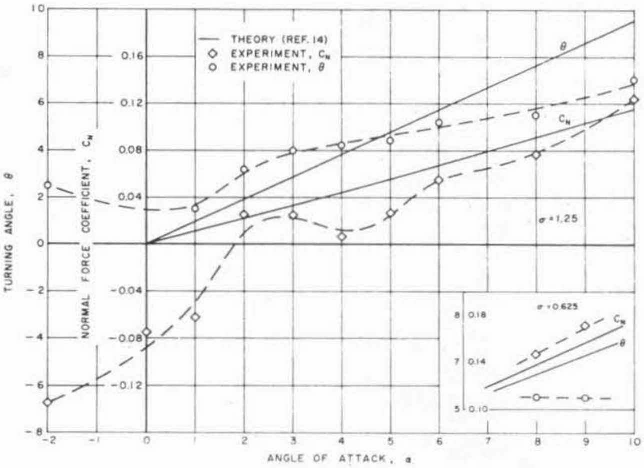

Fig. 10 Comparison of normal force coefficient based on upstream velocity and the turning angle as a function of angle of attack α with theory for fully choked flow in a compressor cascade of solidity $\sigma = 1.25$. The insert shows this comparison for a solidity $\sigma = 0.625$.

It is convenient to consider separately the results for choked flow and those for partial cavitation; we consider first the choked flow as this corresponds to a known limit of cavitating flat plate hydrofoils treated by Betz and Petersohn [14]. These results were used to calculate the theoretical normal force coefficient C_N and turning angle θ for the two cases of the compressor configurations. (The normal force coefficient is the normal force acting on the hydrofoils referred to as the upstream dynamic head.) Fig. 10 illustrates the comparison of these two quantities obtained from theory and from the experiment for the solidity of 1.25. The values of 0.625 solidity are shown in the insert. At the lower angles of attack there is a large difference between observed and calculated value of C_N, but as the angle of attack increases a fairly good agreement is reached for both solidities. This behavior, however, is to be expected since as previously mentioned it was observed that for angles of attack below 6 deg cavitation on the hydrofoils commenced at the leading edge, but on the flat side of the hydrofoil, and for choked conditions cavities also originated from a point about three quarters of the chord downstream from the leading edge on the upper or suction surface of the profile. At higher angles of attack, say 7 deg, cavitation springs from the leading edge and the cavity lies on the normal low pressure surface of the hydrofoil. At the lowest cavitation numbers and higher angles of attack, only the flat surface of the hydrofoil is then wetted and except for a rounded leading edge the flow configuration is similar to that assumed by Betz and Petersohn. Then it is observed that the experimental forces approach reasonably close the predictions in reference [14]. It should be remarked, however, that owing to side wall friction, air coming out of solution, and the geometric limitations of the test section, the minimum cavitation number predicted by potential theory can never be achieved in tests. It can be anticipated therefore that cavitation numbers under conditions of maximum cavity length will exceed these theoretical minimums and that the normal force coefficients will accordingly tend to be somewhat

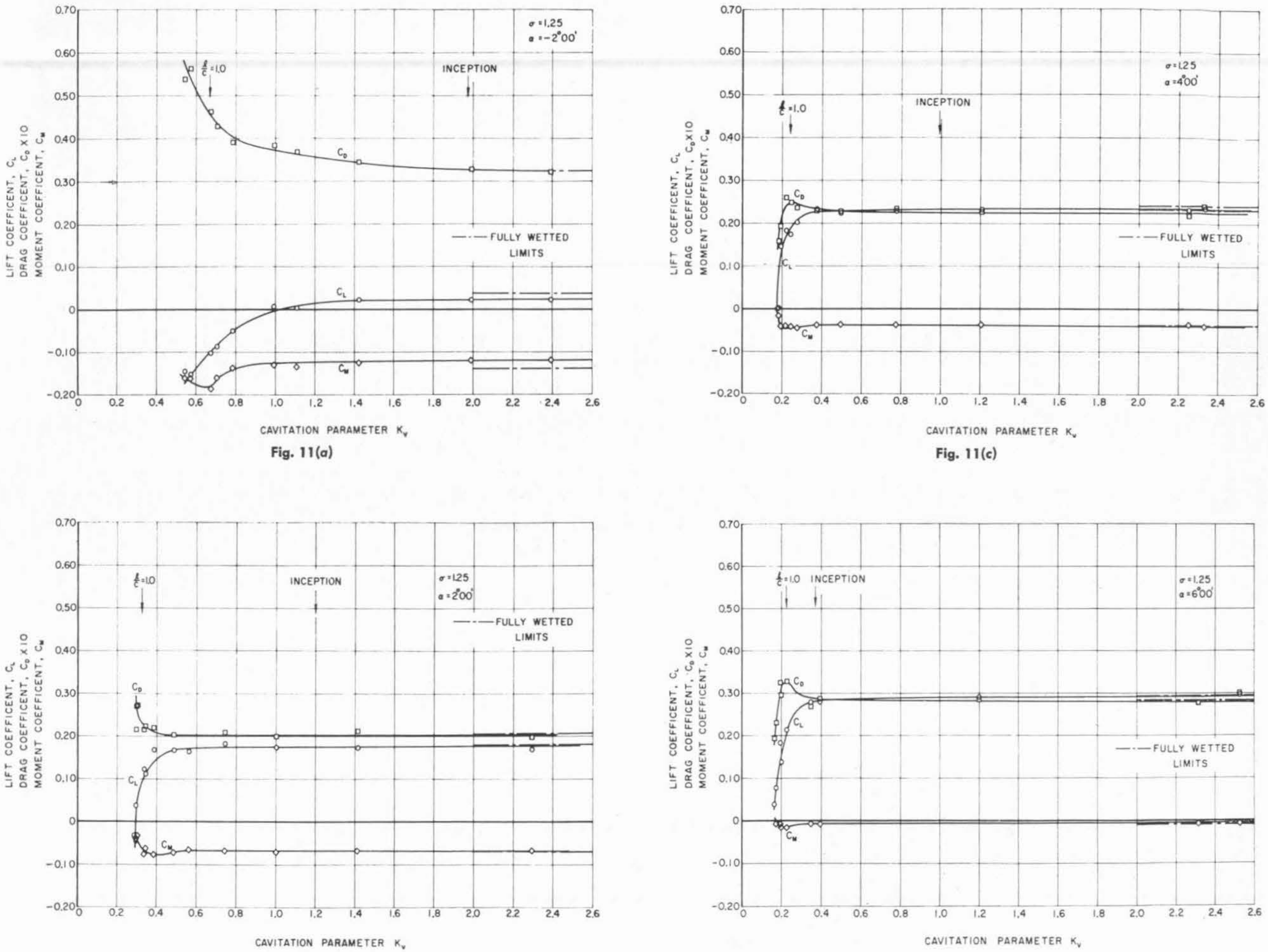

Fig. 11(a)

Fig. 11(c)

Fig. 11(b)

Fig. 11(d)

Fig. 11 Variation of the force coefficients based on upstream velocity with the cavitation parameter K_v for various angles of attack in a compressor cascade of solidity $\sigma = 1.25$

high as is indeed observed for the 0.625 solidity cascade and as is suggested for the other solidity.

The observed flow angles are within two deg of the predicted values for choked flow. As mentioned before, these angles are taken to be the wall angle when the cavitation occurs uniformly on all hydrofoils. The shorter cavities of one and two chords in length are clear and distinct although there is noticeable tip clearance cavitation. Within observational accuracy it was found that the flow angle as determined by uniformity of the cavitation on each of the vanes remained constant when the cavities were longer than about two chords.

We now turn to the remaining portion of the cavitating performance. All these measurements were made with a nominal stagger angle of 45 deg; i.e., a diffusing cascade. The performance of the 1.25 solidity cascade is given in Fig. 11 for various angles of attack and a few representative photographs of the cavitating cascade are shown in Fig. 12. Results for the 0.625 solidity cascade are shown in Figs. 14 and 15. The general behavior of all the graphs is seen to be similar. During the region of partial cavitation the force coefficients remain virtually constant, there being a slight initial rise in lift for the 0.63 solidity case. On decreasing the cavitation parameter further, however, there is a gradual drop in the lift coefficient, with a corresponding rise in drag and a change in moment. For larger angles of attack there is then a subsequent decrease in drag. As the angle of attack is increased, these changes in the force coefficients become more sudden. The fully wetted values are also shown on these graphs as is the point of cavitation inception and the point where the cavity becomes equal to the chord length.

The behavior of the cavitating cascade may be contrasted with that of the corresponding isolated hydrofoil [13]. There, it was found that initially the lift coefficient increased with decreasing cavitation number and there was a corresponding increase in drag. Subsequently with further decrease in cavitation number both lift and drag coefficients decreased, similar to the performance of the 0.625 solidity cascade. The difference in behavior between the isolated hydrofoil and the lower solidity cascade on the one hand, and the 1.25 solidity cascade on the other, is the result of the location of the cavitating region which is different in the two cases. The rise in lift coefficient owing to cavitation on the isolated hydrofoil (and 0.625 solidity cascade) occurs when cavitation starts at the leading edge on the suction side of the profile; the effective camber and hence lift of the hydrofoil is thereby increased. However, the cavitation does not occur in this way in the 1.25 solidity cascade for angles less than 6 deg.

It may be mentioned that comparisons of the experimental results for the supercavitating region with the theoretical results of Cohen and Sutherland [15] show the same trend; however, quantitative comparison could not be made until more results are obtainable from their theory.

From photographs similar to those shown in Fig. 12, cavity length measurements were obtained. Fig. 13 illustrates the behavior of the cavitation parameter K_v with cavity length-to-

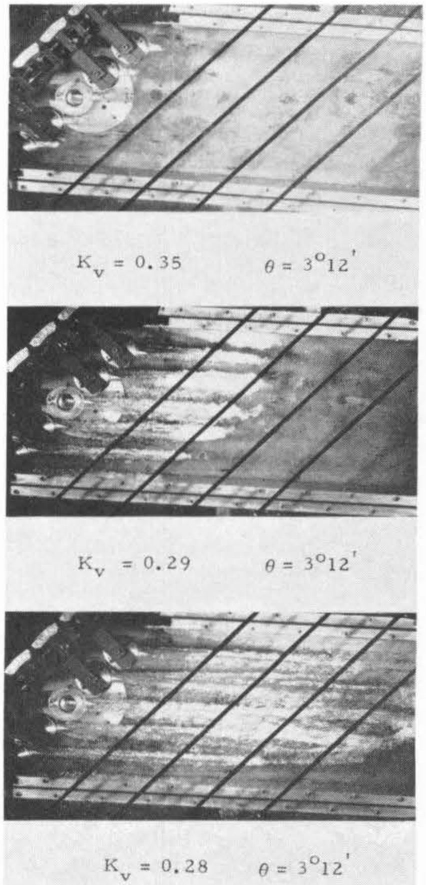

Fig. 12 Cavitation occurring in a compressor cascade of solidity σ = 1.25 for various cavitation parameters K_v at an angle of attack of 2 deg. The diagonal black lines are markers 6 in. apart pasted on the lucite window.

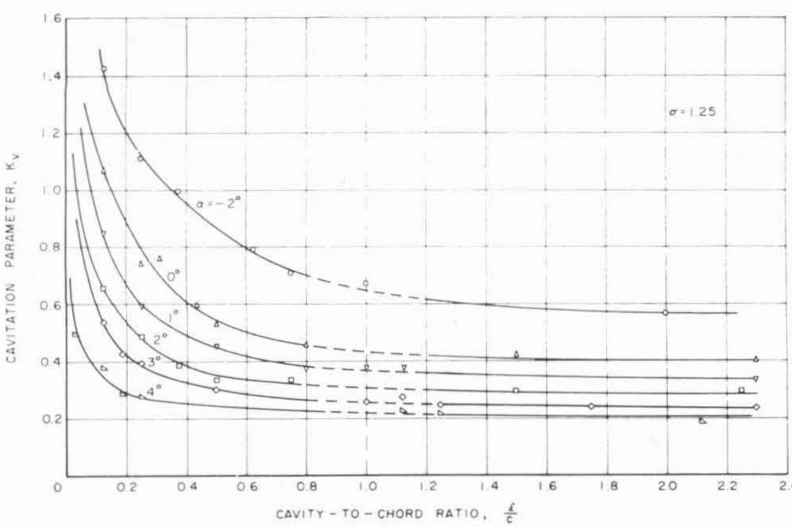

Fig. 13 Cavitation parameter K_v as a function of the length of cavity to chord ratio for various angles of attack in a compressor cascade of solidity σ = 1.25

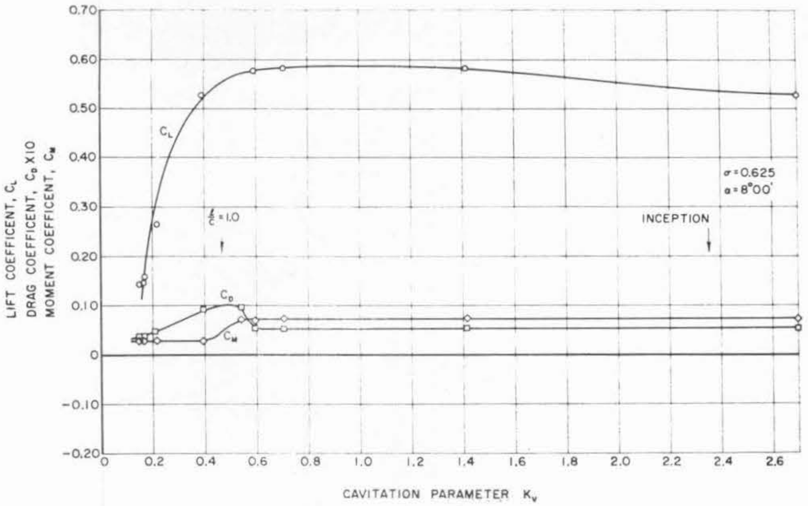

Fig. 14(a)

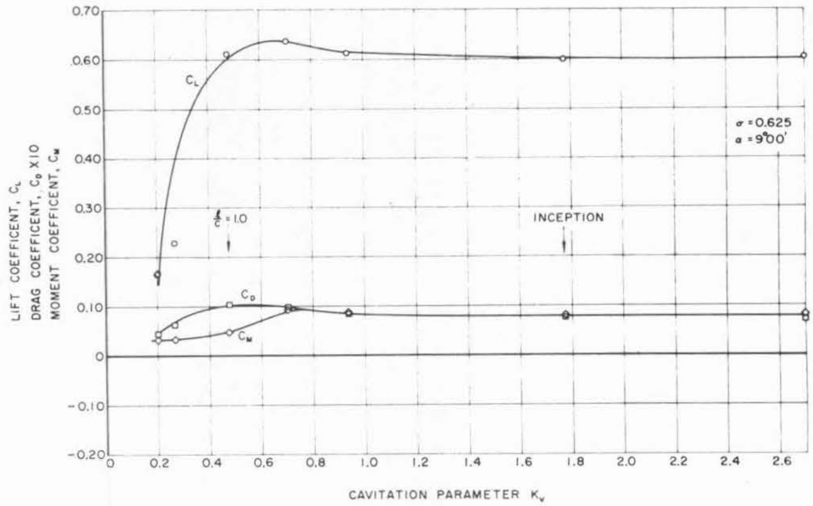

Fig. 14(b)

Fig. 14 Variation of the force coefficients based on upstream velocity with the cavitation parameter K_v for various angles of attack in a compressor cascade of solidity σ = 0.625

chord ratio for a few angles of attack. The region spanning the chord length has been dotted to represent the unsteady region. From these graphs one of the basic experimental difficulties in carrying out these cavitating cascade tests is brought out. It will be noticed that for angles of attack larger than 2 deg the cavitation parameter reaches an asymptotic value fairly rapidly, usually before l/c = 0.5. From then on the smallest change in tunnel pressure will cause the cavity lengths to change enormously. For example, at 30 ft/sec, a typical tunnel speed, a pressure change of approximately <0.3> psi (about <15> mm Hg) will cause the cavity to go from partial cavitation to the choked condition. As can be imagined, it is extremely difficult to obtain repeatable data in this region.

Fig. 16 gives some values of cavity length measurements for the 0.625 solidity cascade. As a matter of interest predicted cavity lengths obtained from the linearized free streamline theory in reference [16] are also shown for the case of 0.5 solidity and angle of attack of 6 deg. Although the cascade parameters are not quite the same, it is interesting to note that these experimental results and predicted lengths are of the same order of magnitude.

It was not possible, unfortunately, to make an extended study of the dynamic properties of the unsteady partial cavitation. It does appear, however, that unsteady cavitation takes place in cascades as it indeed does in many cavitating turbomachines. These oscillations are not small and they give rise to forces which cause severe vibration of the hydrofoils themselves and of the surrounding structure. A few preliminary observations were, however, recorded that may be of interest to report here: A pressure transducer was mounted a few chords upstream of the cascade axis. The frequency of oscillation determined from static pressure fluctuations when they were strongest was 11.8 cps at a tunnel speed of 28 ft/sec. The dimensionless Strouhal number based on chord length was therefore fc/V_1 = 0.14, which is similar to that found for oscillating cavitation on an isolated hydrofoil [13]. The amplitude of the pressure oscillations amounted to

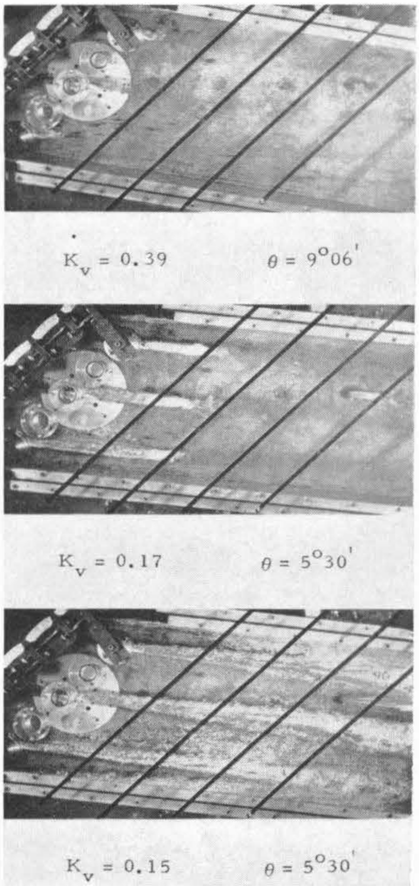

Fig. 15 Cavitation occurring in a compressor cascade of solidity $\sigma = 0.625$ for various cavitation parameters K_v at an angle of attack of 8 deg. The diagonal black lines are markers 6 in. apart pasted on the lucite window.

about one half the approaching velocity head which is thought to be rather large.

A few high-speed motion pictures were taken of the oscillations on the 0.625 cascade. As there are only three vanes in the cascade no real conclusions could be drawn as to how a more proper cascade would behave. These photographs showed that there was some time lag in the development of oscillating cavitation from one vane to the next but because of the limited number of vanes no definite conclusions can now be made on this interesting possibility.

Summary

A series of experiments on the fully wetted and cavitating flow past a cascade of planoconvex hydrofoils was carried out with a nominal stagger angle of 45 deg and solidities of 1.25 and 0.625. It was found that as for the isolated hydrofoil the cavitating performance could be divided into three regions: Partial cavitation, full cavitation or supercavitation, and a zone of unsteady oscillating cavitation separating these two. The fully wetted flows and fully cavitating or choked flows agreed fairly well with existing theoretical guides. Although the effects of side wall boundary layers and related viscous effects were not investigated, it is believed that these were relatively small for the configurations investigated. As a result it is suggested that cavitation investigations of cascades of modest stagger angle can be usefully carried out in water tunnels with rather simple instrumentation. Fluctuating static pressures upstream of the cascade measured in the unsteady region were found to be about one half the approaching velocity head. Such fluctuations in complete turbomachines could presumably occur.

Acknowledgment

This work was supported by the Office of Naval Research under Contract Nonr 220(24) and the Bureau of Naval Weapons under Contract N123(60530)34767A.

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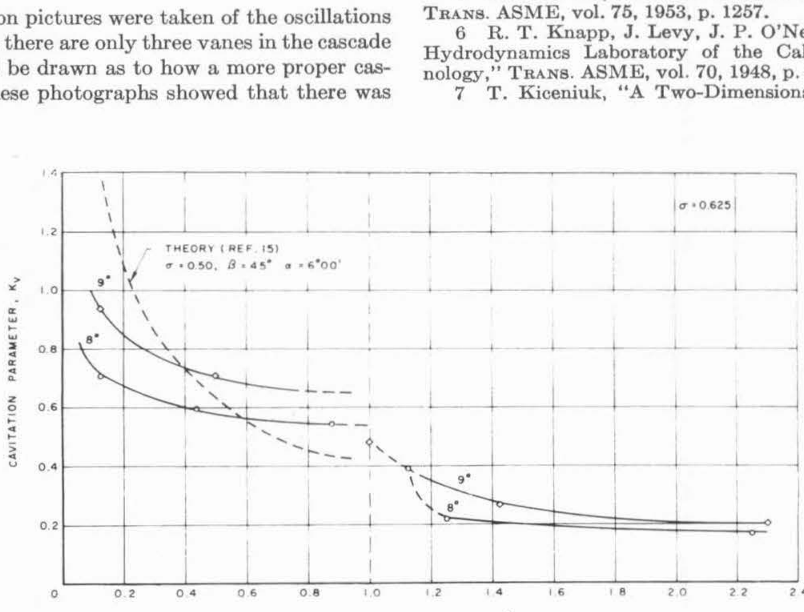

Fig. 16 Cavitation parameter K_v as a function of the length of cavity to chord ratio for two angles of attack in a compressor cascade of solidity $\sigma = 0.625$

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DISCUSSION

F. Numachi[2]

The authors ought to be congratulated because their paper is significant in showing us a threshold of experiments on the fully cavitating flow past a cascade of hydrofoils.

In their text and in its summary, it is stated that the fully wetted flows and fully cavitating flows agreed fairly well with the existing theoretical guide, and a comparison between the experimental results obtained as a whole and the theoretical values is presented. Is this presentation made for the interest from a hydromechanical standpoint or a practical application in view so that calculation of practical values may be made from theoretical values?

If it is made for the first reason, the following should be taken into account: The experimental values of C_{Lm} against its theoretical values in Table 2 have differences of about 10 percent when $\sigma = 1.25$ and 9 percent when $\sigma = 0.65$. It can be surmised from primary research [17][3] into an isolated aerofoil profile that discrepancies to that extent do exist. It is doubtful that it can be appropriately asserted that they "agreed fairly well" with those discrepancies.

If the presentation is made for the second reason, it is conclusively deemed to be difficult to attain. The discrepancies are indicated by our experiments [18]–[24] on cavitation performance on hydrofoil suitable for cascade, shown in Table 2. This leads to the following considerations:

(1) All these hydrofoil profiles have been designed from theoretical calculations so as to have favorable distribution of surface pressure in cascade arrangement, that is, they are considered to have smoother flow past a cascade than plano-convex profiles. In spite of the foregoing fact, some of the differences between experimental values and theoretical values are considerably large.

(2) The discrepancy is small ($Ca_{\mathrm{exp}}/Ca_{\mathrm{th}}$: large) in the case of a profile for decelerating cascade flow, i.e., NAS 10368, which has an excellent performance (small drag-lift ratio at fully wetted flow, late in cavitation inception as well as cavitation stall [25]). But in another profile for accelerating cascade flow, i.e., TNA 16168, which also has an excellent performance, the discrepancy is remarkably large. Consequently, a rule to calculate experimental values from theoretical values of lift has not yet been discovered. The hydrofoil surfaces were finished [25, 26] to be 0.2 S (Japan Industrial Standard) or 2μ in ASA; the tolerance of the specimen dimensions is ± 0.01 mm for 100 mm chord length.

Table 2 Theoretical and experimental values of lift coefficient at incidence angle $\alpha_{\infty\,\mathrm{th}}$

Hydrofoil	Ca_{th}	$\alpha_{\infty\,th}$	Ca_{exp}	Ca_{exp}/Ca_{th}
N.A.S. 10368 (18)	0,60	2,9°	0,595	0,995
N.A.S. 12368 (18)	0,60	1,33°	0,545	0,91
N.A.S. 13368 (18)	0,60	2,5°	0,565	0,94
N.A.S. 14368 (18)	0,60	2,5°	0,565	0,94
N.A.S. 11268 (20)	0,60	2,17°	0,555	0,925
N.A.S. 14268 (20)	0,60	2,15°	0,50	0,835
N.A.S. 10168 (21)	0,60	1,65°	0,565	0,94
N.A.S. 12168 (20)	0,60	1,75°	0,56	0,93
T.N.A. 10368 (22)	0,60	−1,08°	0,57	0,95
T.N.A. 11368 (22)	0,60	−1,12	0,54	0,90
T.N.A. 13168 (22)	0,60	−1,58°	0,58	0,965
T.N.A. 14168 (22)	0,60	−1,25°	0,58	0,965
T.N.A. 13268 (23)	0,60	−0,27°	0,585	0,975
T.N.A. 14268 (23)	0,60	−0,12°	0,595	0,99
T.N.A. 15268 (23)	0,60	−0,88°	0,50	0,835
T.N.A. 12368 (24)	0,60	−1,05°	0,598	0,995
T.N.A. 14368 (24)	0,60	−0,83°	0,590	0,985
T.N.A. 15168 (24)	0,60	−0,87°	0,520	0,87
T.N.A. 16168 (24)	0,60	−1,28°	0,500	0,835
T.N.A. 17168 (24)	0,60	−1,36°	0,495	0,825

Ca_{th} and Ca_{exp} : Theoretical and experimental values of lift coefficient at incidence angle $\alpha_{\infty\,th}$.
$\alpha_{\infty\,th}$: Value of incidence angle corresponding theoretically to Ca_{th}

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[2] Professor Emeritus, Institute of High Speed Mechanics, Tōhoku University, Sendai, Japan. Mem. ASME.

[3] Numbers in brackets designate Additional References at end of discussion.

I. S. Pearsall[4]

The experiments described are the first published on a cascade of supercavitating hydrofoils, and for this reason the paper is of great value and interest. The experimental techniques involved in testing such a cascade under cavitating conditions are most difficult and the successful outcome described in the paper emphasizes the care and thought put into the investigation. In view of the increasing use of hydrofoils in part-cavitating and supercavitating conditions in pumps and propellers, the results are particularly timely. It is only a pity that the investigation was so limited, and that sections other than the plano-convex section have not been tested, as well as the higher stagger angles used in cavitating pumps.

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First, I would like to call attention to two minor mistakes. In Fig. 14, the values of C_D appear to be on the same scale as C_L and not the 10 times scale given (used also in Fig. 11). The definition of nominal stagger angle γ in Fig. 5 does not agree with the Nomenclature. The angle marked γ in Fig. 5 is actually $90-\gamma$. The authors are not very clear in their definition of stagger angles. On p. 694 it is stated that nominal stagger angle (γ) is 45 deg and this is what one would expect. However, in Table 1 and Fig. 16, the reader appears to be asked to compare experimental results for $\gamma = 45$ deg, with theoretical in which $\beta = 45$ deg.

Cascade data of this type would be of great assistance in the

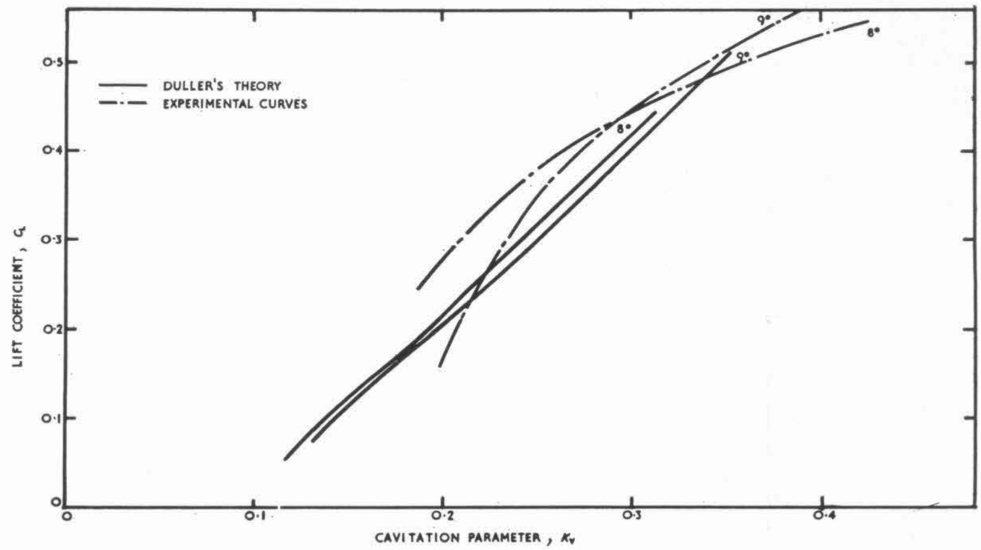

Fig. 17 Variation of lift coefficient with cavitation parameter compressor cascade $\gamma = 45$ deg, $\sigma = 0.625$, $\alpha = 8$ deg, 9 deg

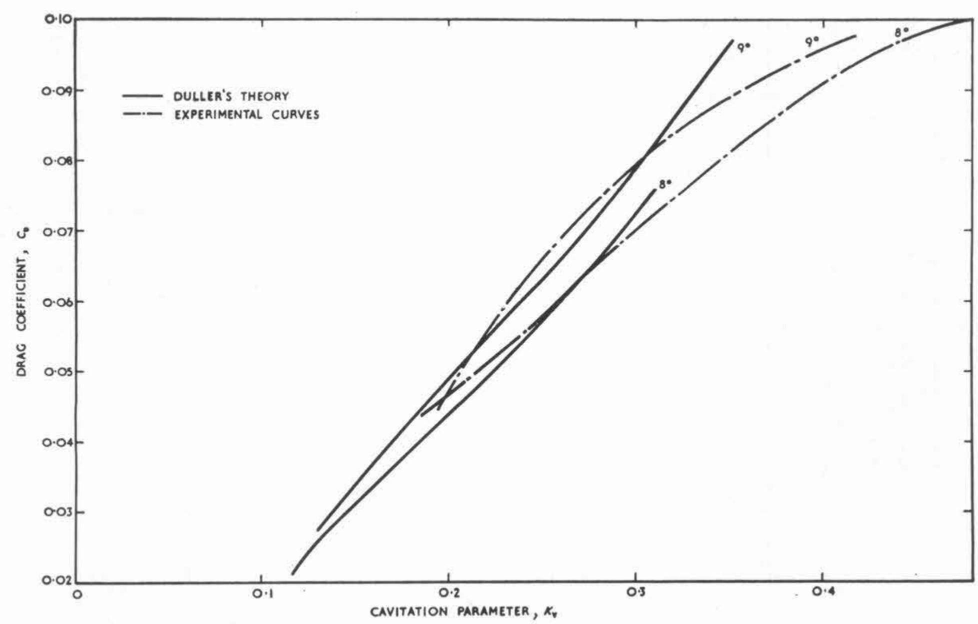

Fig. 18 Variation of drag coefficient with cavitation parameter compressor cascade $\gamma = 45$ deg, $\sigma = 0.625$, $\alpha = 8$ deg, 9 deg

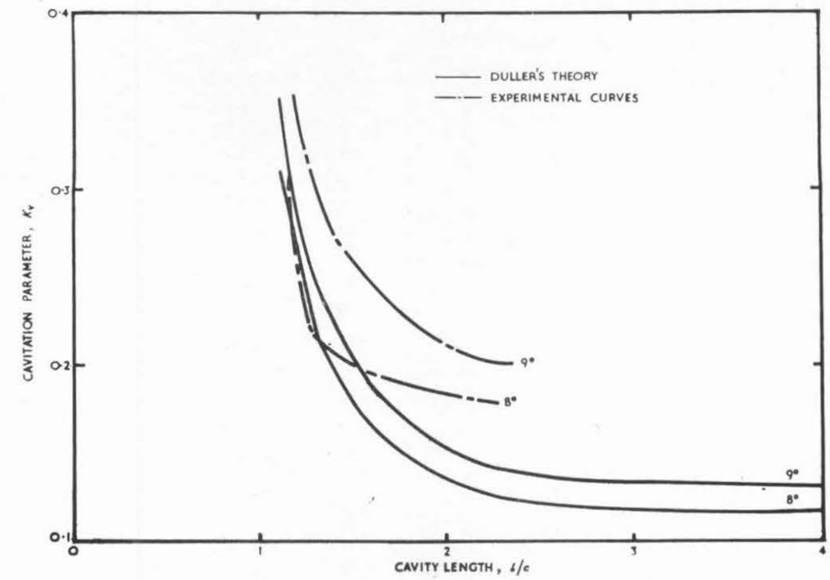

Fig. 19 Variation of cavity length with cavitation parameter compressor cascade $\gamma = 45$ deg, $\sigma = 0.625$, $\alpha = 8$ deg, 9 deg

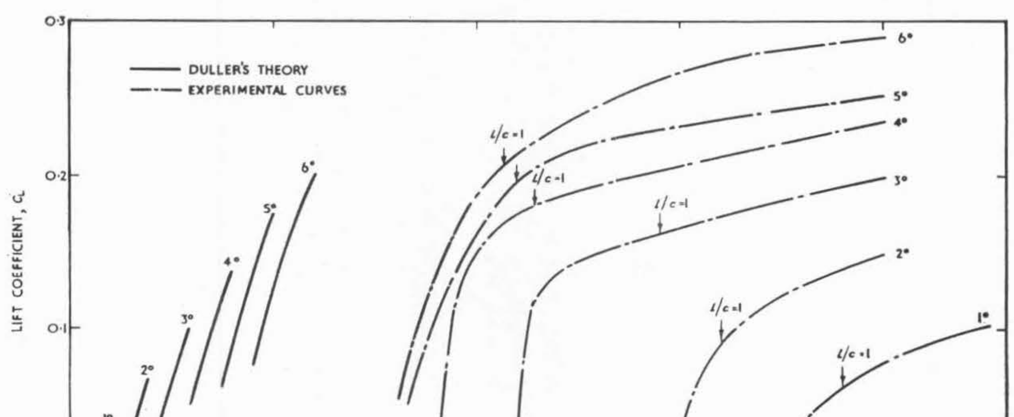

Fig. 20 Variation of lift coefficient with cavitation parameter compressor cascade $\gamma = 45$ deg, $\sigma = 1.25$, $\alpha = 1$ deg–6 deg

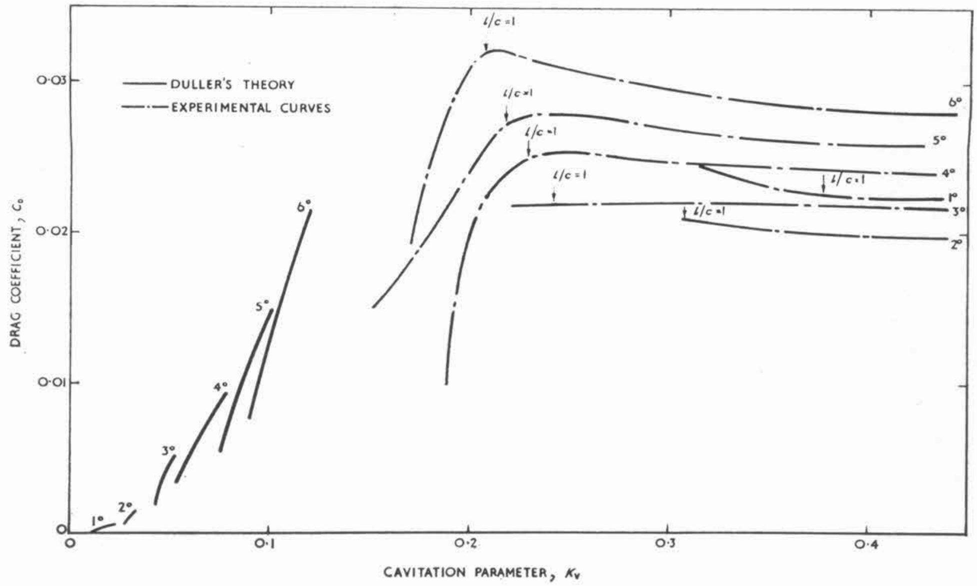

Fig. 21 Variation of drag coefficient with cavitation parameter compressor cascade $\gamma = 45$ deg, $\sigma = 1.25$, $\alpha = 1$ deg–6 deg

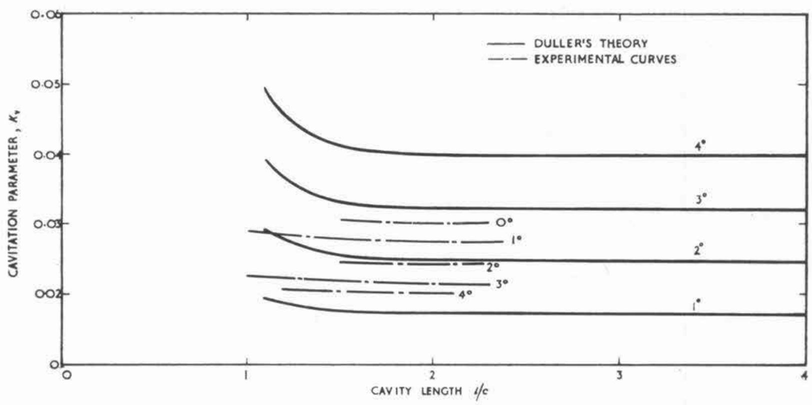

Fig. 22 Variation of cavity length with cavitation parameter compressor cascade $\gamma = 45$ deg, $\sigma = 1.25$, $\alpha = 1$ deg to 4 deg

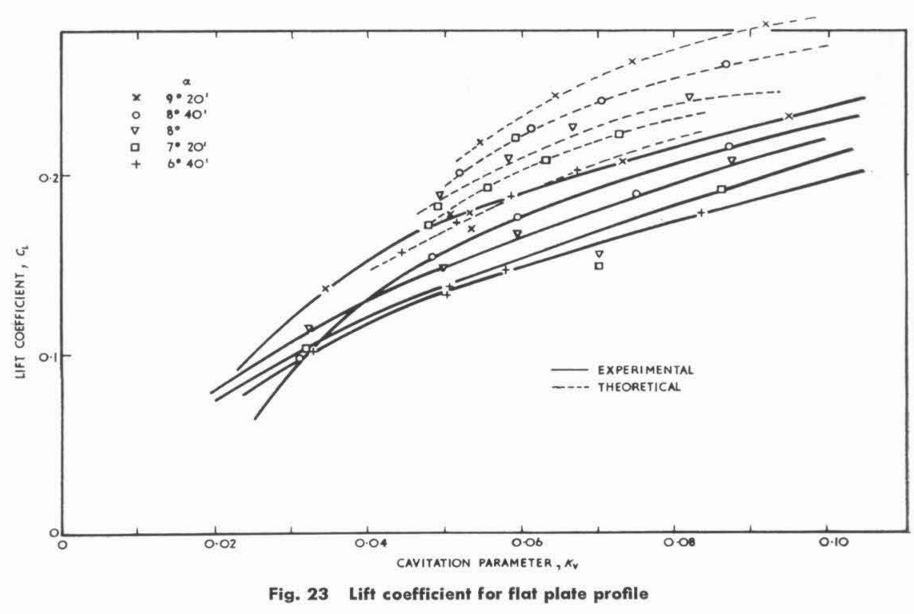

Fig. 23 Lift coefficient for flat plate profile

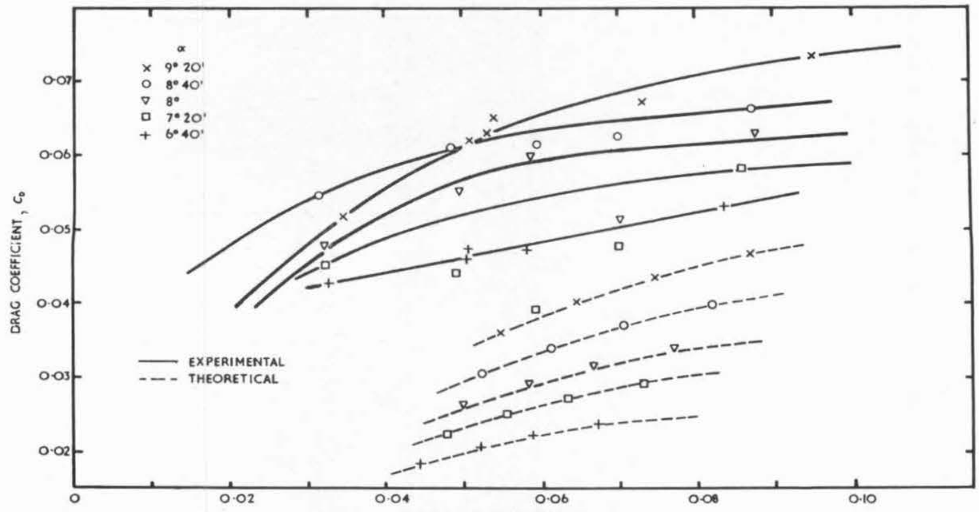

Fig. 24 Drag coefficient for flat plate profile

design of cavitating pumps at present being studied at NEL. Unfortunately, the stagger angles used by the authors are too low to be directly applied, but it is of interest to compare the authors' results with theoretical methods developed at NEL.

The basis of these methods is a linearized theory of two-dimensional supercavitating cascade flow developed by Duller[5] as an extension of the work of reference [15], to general shapes of hydrofoils. Further extension to the cases of part-cavitation is being analyzed, but the comparisons shown in Figs. 17–22 are for the supercavitating case; that is, cavity starting at the leading edge and having a minimum length equal to the chord. Figs. 17–19 show results for a cascade of solidity 0.625, and correspond to Figs. 14 and 16, in the authors' paper. The agreement in this case between theory and the authors' experimental results is good. With the higher solidity of 1.25, Figs. 20–22, corresponding to Figs. 11 and 13 in the paper, it is seen that the assumptions of the linearized theory fail completely to predict performance at these choked conditions. At the two conditions shown, the solidity is so high that the cascade is choking and cavity length is very sensitive to cavitation number. It is felt that the potential flow approach of Duller[5] will give better representation of these flows at lower solidities. As confirmation of this, Figs. 23 and 24 show comparisons between theoretical values and experimental values of lift and drag. The latter were obtained from mean values across the blade on a supercavitating pump running at 2500 rpm. The stagger angle β was 68 deg and the solidity 0.36. Incidence varied from 6 deg to 9 deg. In this case, the theoretical drag is considerably below that obtained by experiment, as the latter includes the secondary flow losses.

All these results illustrate the urgent need for more cascade data on part and fully cavitating hydrofoils. The work described in the paper makes a valuable start.

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Reprinted from the December 1967
Journal of Basic Engineering